

# Stress concentration around holes (International series of monographs in aeronautics and astronautic

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## Recent General Solutions in Linear Elasticity and Their Applications

*A review is given on the progress in the study of general solutions of elasticity and their applications since 1972. Apart from summarizing and remarking the development of the general solution method in literature, this review aims to present the readers with a systematic and constructive scheme to develop general solutions from given governing differential equations and then to prove their completeness and investigate their nonuniqueness features. The effectiveness of the constructive scheme manifests itself in the fact that almost all the classic solutions, including not just classic displacement potentials but also classic stress functions, can be rederived by using this scheme. Furthermore, thanks to the systematic features of the scheme, it produces a constructive approach to study the completeness and nonuniqueness of general solutions and possesses more flexibility, which facilitates the extension of classic general solution methods to more general systems governed by elliptic differential equations. Under the framework of this scheme, a review is presented on wide applications of general solutions in a variety of research areas, ranging from problems with different materials, isotropic or anisotropic, to various coupling problems, such as thermoelasticity, magnetoelasticity, piezoelectric elasticity, porous elasticity, and quasicrystal elasticity, and to problems of different engineering structures, for instance, the refined theories for beams and plates. There are 213 references cited in this review article. [DOI: 10.1115/1.2909607]*

### 1 Introduction

Since the basic governing equations for linear elasticity were established by Cauchy, Navier and Poisson in the 19th century, studies on analytic solutions of these equations have become a core issue in elasticity. Among the most significant contributions are the Saint Venant method for beam bending problems and the Muskhelishvili solution [1] for the generalized two-dimensional (2D) problems based on the complex variable function method. Later on, many researchers extended their interests to seeking solutions for three-dimensional (3D) problems.

After decades of effort by researchers, the general solution method in elastic theory has been proven a powerful tool in solving 3D problems. Previous reviews on the development history of the elastic general solutions have been done by Sternberg [2], Teodorescu [3,4], and Gurtin [5]. In Gurtin's book, the completeness of two most famous general solutions, i.e., the Boussinesq-Galerkin solution and the Papkovitch-Neuber solution, received discussion. As the history of the development informs us, because no systematic method was available, the accomplishment of the constructibility and the completeness and nonuniqueness analysis of those classic general solutions required strong sense of mathematic propensity possessed usually by few researchers, so that the work on general solutions were done simply case by case and restricted mainly on classic elasticity problems.

In the past three decades, a great deal of achievement has been further obtained, which appears mainly in three aspects. One major contribution is the development of a unified and constructive scheme to derive general solutions for anisotropic elasticity. This scheme provides people with a manifest way to pursue general solutions once they have the governing equations for the problems in question. Meanwhile, thanks to its constructive feature in derivation, the scheme spontaneously offers the proof of completeness of the general solutions obtained. The unified scheme proposes

also a uniform method for investigating the nonuniqueness and the degree of nonuniqueness of general solutions.

Another improvement occurs with the fact that general solutions have been receiving a wide range of applications in boundary value problems. For instance, general solutions play a key role in solving well-known Boussinesq's problem and Mindlin's problem and in analyzing fracture problems. There are also good examples in the study of 2D elastic strips and 3D elastic layers, which in return give rise to the refined theory of beams and plates.

The third achievement is that the general solution concept and related methods have been broadly adopted to analyze general linear differential equations. An illustrative example is the development of stress functions—such as the Beltrami-Michler stress function—which fundamentally are general solutions of the equilibrium equations in terms of stresses. Further examples are found in applying the elastic general solution theory to a variety of new disciplinary branches or interdisciplinary fields burgeoning and developing during the second half of the 20th century—such as thermoelasticity, magnetoelasticity, piezoelectric elasticity, porous elasticity, and quasicrystal elasticity—branches and fields which are largely characterized by taking into account multifield coupling effects.

It is the very purpose of this review to present the readers with these progresses—the unified scheme for general solution study, the application of general solutions obtained, and the generalization of the general solution concept—in a manner which includes comparison with classical results in the history and casts light on potential development in the future. Besides this introductory section and a concluding section at the end, the main body of this article is divided into three sections, which correspond to each aspect of the achievement.

To begin with, Sec. 2 will detail the constructive scheme for the study of elastic general solutions and related results obtained through this scheme. It consists of five subsections. Sec. 2.1 essentially establishes the basis of the present review article. Four theorems will be introduced, which fundamentally outline the constructive scheme for the constructibility and the completeness and nonuniqueness study of general solutions for anisotropic elas-

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Published online: May 7, 2008. Transmitted by ASME, Editor J. Simeonidis.

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Astronautics, Vol. 1. ally organized series of conferences initiated by the 1st Polish Solid materials, mechanics of structures and optimization, shells theory and . Thermal Stresses Around an Interface Rigid Circular Inclusion in a As can be expected from [1], the highest stress concentration is hole has been drilled. Distributed under a Creative Commons Attribution International License In the context of structural mechanics, it is established that the basis . 1. Thin shell structures exhibit significant imperfection sensitivity: (a) faceted show that the stress concentration of  $r_{xx} = 30$  at the edge of the hole (i.e., at.ssdm conf 25th Structures, Structural Dynamics and Materials Volume 1: Solid Earth (SE) aogs 10 Advances in Geosciences, Volume .. Aeronautics and Astronautics, International Meeting and Technical Display on Global Gebrueder Borntraeger Geoexploration Monographs Series BAVSM Berliner. INTERNATIONAL CONFERENCE ON INTERNET COMPUTING, VOLS 1 AND .. 21st International Congress on Instrumentation in Aerospace Simulation Facilities IEEE Asian Solid-State Circuits Conference Proceedings of Technical IMECE MECHANICS OF SOLIDS, STRUCTURES AND FLUIDS, VOL Items - NASA/TM 1 Collected Papers in Structural Mechanics Honoring Dr. Janies .. International Journal of Solids and Structures, Vol. by McDonnell Douglas Aerospace (now Boeing Phantom Works Division). .. stress concentrations and stress and strain distributions around a narrow elliptical.

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